BACCALAURÉAT 2013

Epreuve de Discipline Non Linguistique - Mathématiques/Anglais

Quadratic Equations

The Babylonians, as early as 2000 BC (displayed on Old Babylonian clay tablets) could solve a pair of simultaneous equations of the form:

$$x + y = p \; ; \; xy = q \quad (1)$$

Babylonian mathematicians from circa 400 BC and Chinese mathematicians from circa 200 BC used the method of *completing the square* to solve quadratic equations with positive roots, but did not have a general formula. Euclid, the Greek mathematician, produced a more abstract geometrical method around 300 BC.

In 628 AD, Brahmagupta, an Indian mathematician, gave the first explicit solution of the quadratic equation. Mohammad bin Musa Al-Khwarizmi (Persia, 9th century) developed a set of formulas that worked for positive solutions based on Brahmagupta. The Catalan Jewish mathematician Abraham Bar Hiyya Ha-Nasi authored the first book to include the full solution to the general quadratic equation.

The writing of the Chinese mathematician Yang Hui (1238-1298 AD) represents the first in which quadratic equations with negative coefficients of '*x*' appear, although he attributes this to the earlier Liu Yi. The first appearance of the general solution in the modern mathematical literature is evidently in an 1896 paper by Henry Heaton.

Adapted from Wikipedia and AS Pure Mathematics.

Questions:

(a) Show that (1) is equivalent to a polynomial equation of degree 2.

(b) Given that $x^2 + 10x + 36 = (x + a)^2 + b$, where *a* and *b* are constants, find the value of *a* and the value of *b* (this process is known as *completing the square*).

(c) Hence show that the equation $x^2 + 10x + 36 = 0$ has no real roots.

(d) The equation $x^2 + 10x + k = 0$ has equal roots: find the value of *k*.

(e) For this value of *k*, sketch the graph of $y = x^2 + 10x + k$, showing the coordinates of any points at which the graph meets the coordinates axes.